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Exact solution of the six-vertex model with domain wall boundary conditions. Antiferroelectric phase

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1 Abstract

This is a joint work of the author with Karl Liechty, and it is a continuation of the works [1] of Bleher and Fokin and [2], [3] of Bleher and Liechty. In the works [1], [2] and [3] the large n asymptotics is obtained for the partition function Z_n of the six-vertex model with domain wall boundary conditions in the disordered phase region, the ferroelectric phase region, and on the critical line between these two regions. In the present paper we obtain the large n asymptotics of Z_n in the antiferroelectric phase region, with the weights $a = \sinh(\gamma - t)$, $b = \sinh(\gamma + t)$, $c = \sinh(2\gamma)$, $|t| < \gamma$. We prove that if the ratio $\frac{t}{\gamma}$ is either rational or satisfies some Diophantine condition, then the partition function has the asymptotic behavior $Z_n = C\theta_4(n\omega)n^{\kappa}F^{n^2}[1 + O(n^{-1})]$ as $n \to \infty$, where the exponent κ is given by two different formulae for rational and Diophantine values of $\frac{t}{\gamma}$. The proof is based on the Riemann-Hilbert approach to the underlying discrete orthogonal polynomials.

References

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- [3] P.M. Bleher and K. Liechty, Exact solution of the six-vertex model with domain wall boundary condition. Critical line between ferroelectric and disordered phases, J. Statist. Phys. 134 (2009), 463–485.