The staircase method: Lax-pairs, integrals and integrating factors of OΔE's.

Omar Rojas[1] G.R.W. Quispel[1], P. van der Kamp[1]

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1. Department of Mathematics. La Trobe University. Victoria, 3086 (Australia).

We give Lax-pairs for ordinary difference equations (OΔE's) $f_n = 0$, obtained by (z_1, z_2)-traveling-wave reductions from integrable partial difference equations (PΔE's) $f_{l,m} = 0$ [1]. From the Lax-equation for PΔE's, namely $L_{l,m} M_{l,m}^{-1} - M_{l+1,m}^{-1} L_{l,m+1} = f_{l,m} N_{l,m}$, in which the matrices $L_{l,m}, M_{l,m}, N_{l,m}$ depend on a spectral parameter k and $N_{l,m}$ is nonsingular on the equation [2] we construct, using the standard-staircase method, the *monodromy* matrix $\mathcal{L}_n$ and an *auxiliary* matrix $\mathcal{M}_n$ such that

$$\mathcal{M}_n \mathcal{L}_n - \mathcal{L}_{n+1} \mathcal{M}_n = f_n \mathcal{N}_n$$

holds. It follows that the trace of the monodromy matrix is invariant, that is

$$\mathrm{tr}(\mathcal{L}_{n+1}) - \mathrm{tr}(\mathcal{L}_n) = f_n \Lambda_n,$$

with integrating factor $\Lambda_n = -\mathrm{tr}(N_n M_n \mathcal{L}_n L_n^{-1})$. Taking $z_1 = 1$ the coefficients of the k-expansion of the trace of the monodromy matrix $\mathrm{tr}(\mathcal{L}_n)$, and of the integrating factor Λ_n, are linear in terms of certain multi-sums of products Θ. For general z_1 we expect the invariants to be polynomial in the Θ's of degree z_1.

References

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