Parametric Instability of Coherent States of a Real Scalar Field

Vladimir A. Koutvitsky[1] Eugene M. Maslov[1]

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1. IZMIRAN, 142190 Moscow Region, Troitsk (Russia)

We investigate stability of both localized time-periodic coherent states (pulsons) and uniformly distributed coherent states (oscillating condensate) of a real scalar field satisfying the Klein-Gordon equation with a logarithmic nonlinearity. Such nonlinearities appear, in particular, in some supersymmetric extensions of the Standard Model.

The linear analysis of time-dependent parts of the pulson's perturbation leads to the Hill equation with a singular coefficient. To evaluate the characteristic exponent we extend the Lindemann-Stieltjes method, usually applied to the Mathieu and Lamé equations, to the case that the periodic coefficient in the general Hill equation is an unbounded function of time. As a result, we derive a formula for the characteristic exponent and plot the stability-instability chart. We verify these calculations by the direct numerical integration of the Hill equation. The consideration of the space-dependent parts of the perturbation leads to the Schrödinger equation having a discrete spectrum of localized solutions. In particular, for the nodeless pulsons these solutions are expressed in terms of the Hermite polynomials. Using these results we show that the pulsons of any amplitudes, remaining well-localized objects, lose their coherence with time. This means that, strictly speaking, all pulsons of the model considered are unstable. Nevertheless, for the nodeless pulsons the rate of the coherence breaking in narrow ranges of amplitudes is found to be very small, so that such pulsons can be considered as long-lived.

Further, we use the obtaned stability-instability chart to examine the Affleck-Dine type condensate oscillating around the minimum of the logarithmic potential. We estimate the wavenumber and the oscillation amplitude for which the characteristic exponent is maximal and discuss the fragmentation processes. We conclude that, due to the parametric instability, the oscillating condensate can decay into an ensemble of the nodeless pulsons.