Integrable maps and Diophantine equations

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By a method originally due to Hirota, many well known integrable PDEs can be rewritten in terms of an holomorphic tau-function, and this holds for partial difference equations such as the Hirota bilinear difference equation (discrete KP). For birational maps that satisfy the singularity confinement property, it seems that there should also be an analogue of the tau-function (even in non-integrable cases). This tau-function satisfies a difference equation that displays the Laurent phenomenon i.e. all iterates are Laurent polynomials in the initial data. In the integrable case, rational invariants lead to associated Diophantine equations. This is illustrated with the example of a third-order recurrence that generates infinitely many integer points on a pencil of cubic surfaces. This generalizes an old result of Gauss concerning integer points on conics.