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A New Extended DKP Hierarchy and Generalized Dressing Approach

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(with Xiaojun Liu)

Let $L = \Delta + f_0 + f_1\Delta^{-1} + f_2\Delta^{-2} + \cdots$, $B_n = L^n_+$, the DKP hierarchy reads[1,2]

$$L_{t_n} = [B_n, L].$$

Inspired by the squared eigenfunction symmetry constraint [3], we propose
Proposition 1. The extended DKP hierarchy (exDKPH) is constructed as follows

$$L_{t_n} = [B_n, L] \tag{1a}$$

$$L_{\tau_k} = [B_k + \sum_{i=1}^{N} \psi_i \Delta^{-1} \phi_i, L] \tag{1b}$$

$$\psi_{i,t_n} = B_n(\psi_i),\ \phi_{i,t_n} = -B_n^*(\phi_i),\ i = 1, \cdots, N. \tag{1c}$$

Under (1c), the commutativity of (1a) and (1b) leads to the zero curvature equation for the exDKPH

$$B_{n,\tau_k} - (B_k + \sum_{i=1}^{N} \psi_i \Delta^{-1} \phi_i)_{t_n} + [B_n, B_k + \sum_{i=1}^{N} \psi_i \Delta^{-1} \phi_i] = 0, \tag{2a}$$

$$\psi_{i,t_n} = B_n(\psi_i),\ \phi_{i,t_n} = -B_n^*(\phi_i),\ i = 1, 2, \cdots, N. \tag{2b}$$

and its Lax representation

$$\Psi_{t_n} = B_n(\Psi), \quad \Psi_{\tau_k} = (B_k + \sum_{i=1}^{N} \psi_i \Delta^{-1} \phi_i)(\Psi). \tag{3}$$

(2) contains two types of DKP equation with self-consistent sources(DKPESCS).

The t_n-reduction of the exDKPH leads to

$$B_{n,\tau_k} = [(B_n)_+^{\frac{k}{n}} + \sum_{i=1}^{N} \psi_i \Delta^{-1} \phi_i, B_n], \tag{4a}$$

$$B_n(\psi_i) = \lambda_i^n \psi_i,\ B_n^*(\phi_i) = \lambda_i^n \phi_i,\ i = 1, 2, \cdots, N. \tag{4b}$$

The τ_k-reduction leads to

$$(B_k+\sum_{i=1}^N\psi_i\Delta^{-1}\phi_i)_{t_n}=[(B_k+\sum_{i=1}^N\psi_i\Delta^{-1}\phi_i)_+^{\frac{n}{k}},B_k+\sum_{i=1}^N\psi_i\Delta^{-1}\phi_i], \tag{5a}$$

$$\psi_{i,t_n}=(B_k+\sum_{i=1}^N\psi_i\Delta^{-1}\phi_i)_+^{\frac{n}{k}}(\psi_i), \tag{5b}$$

$$\phi_{i,t_n}=-(B_k+\sum_{i=1}^N\psi_i\Delta^{-1}\phi_i)_+^{\frac{n}{k}*}(\phi_i),\ i=1,2,\cdots,N. \tag{5c}$$

Assume that L of exDKPH can be written in the dressing form: $L=W\Delta W^{-1}$. In this paper, we propose a generalized dressing approach for solving the exDKPH .

Proposition 2. Let f_i, g_i satisfy

$$f_{i,t_n}=\Delta^n(f_i),\quad f_{i,\tau_k}=\Delta^k(f_i)\quad(i=1,\ldots,N) \tag{6a}$$

$$g_{i,t_n}=\Delta^n(g_i),\quad g_{i,\tau_k}=\Delta^k(g_i). \tag{6b}$$

$$h_i=f_i+\alpha_i(\tau_k)g_i\quad i=1,\ldots,N \tag{7}$$

$$W=\frac{1}{Wrd(h_1,\cdots,h_N)}\begin{vmatrix} h_1 & h_2 & \cdots & h_N & 1\\ \Delta(h_1) & \Delta(h_2) & \cdots & \Delta(h_N) & \Delta\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \Delta^N(h_1) & \Delta^N(h_2) & \cdots & \Delta^N(h_N) & \Delta^N \end{vmatrix} \tag{8}$$

$$Wrd(h_1,\cdots,h_N)=\begin{vmatrix} h_1 & h_2 & \cdots & h_N\\ \Delta(h_1) & \Delta(h_2) & \cdots & \Delta(h_N)\\ \vdots & \vdots & \vdots & \vdots\\ \Delta^{N-1}(h_1) & \Delta^{N-1}(h_2) & \cdots & \Delta^{N-1}(h_N) \end{vmatrix} \tag{9}$$

$$\psi_i=-\dot{\alpha}_iW(g_i)\quad \phi_i=(-1)^{N-i}\frac{Wrd(\Gamma h_1,\cdots,\hat{\Gamma h_i},\cdots,\Gamma h_N)}{Wrd(\Gamma h_1,\cdots,\Gamma h_N)},i=1,\ldots,N \tag{10}$$

then W, L, ψ_i and ϕ_i satisfy (1).

By using Proposition 2, some soliton solutions for two types of DKPESCS are obtained.

References

[1] K. M. Tamizhmani and S. Kanaga Vel, *Chaos Soliton and Fractal*, **11**, 137 (2000).

[2] J. S. He , S. W. Liu and Y. Cheng. The determinant representation of the gauge transformation for discrete KP hierarchy, preparation.

[3] W.Oevel. Darboux transformation for integrable lattice system, Nonlinear Physics. Theory and Experiment, 233-240, E. Alfinito, L.Martina and F. Pempinelli(eds), World Scientific, Singapore (1996).