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Bifurcation and attractors of double-diffusive convection in plain layer

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Double-diffusive convection between two horizontal planes is investigated. Sequence of bifurcations from stationary motions to stochastic motions is demonstrated. We found that the attractor has the structure of a Mobius band in chaotic regimes. With the help of Poincare sections and Poincare maps we show modifications of the attractor with the increase of supercriticality. First, Poincare map can be represented as a one-valued function. With the growth of supercriticality Poincare map remains one-dimensional but now it has many minima and self-intersections so it can't be approximated by some function. With the help of Lyapunov exponents we show the divergence of trajectories on the attractors. Relative residual of the initial Navier-Stokes equations is calculated for all the numerical solutions, so we can affirm that the numerical solutions almost exactly represent the genuine solution (the third order of accuracy) and properties of the attractor adequately correspond to the initial model. The convergence of Bubnov-Galerkin method is demonstrated with the help norms of kinetic energy and dissipation function.

Despite the great development of the nonlinear sciences since the works of Lorenz, Ruelle and Takens not so many works are devoted to investigation of transition to stochastic modes with full Navier-Stokes equations compared with the truncated models. Here the method of Bubnov-Galerkin will be used and the number of the base functions will be taken so that the relative residual to be of the third order of smallness, so we can affirm that the numerical solution represents the genuine solution. The traditional geometry in which convective motions have been quantitatively analyzed [1] confines the fluid between two infinite horizontal planes, heated, and in this case also salted, from below.

We can write the governing Boussineq equations of motion as:

$$\frac{1}{\sigma}\Delta\frac{\partial\psi}{\partial t}+R_T\frac{\partial\tau}{\partial x}-R_S\frac{\partial s}{\partial x}-\Delta^2\psi=\frac{1}{\sigma}J(\psi,\Delta\psi), \tag{1}$$

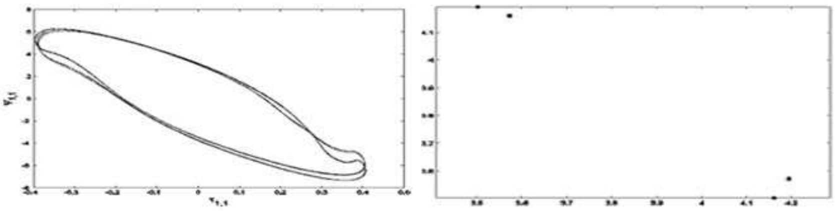
$$\frac{\partial\tau}{\partial t}+\frac{\partial\psi}{\partial x}-\Delta\tau=J(\psi,\tau), \tag{2}$$

$$\frac{\partial s}{\partial t}+\frac{\partial\psi}{\partial x}-\kappa\Delta s=J(\psi,s). \tag{3}$$

$T=T_0+(T_1-T_0)(1-z+\tau)$, $\quad S=S_0+(S_1-S_0)(1-z+s)$. The full system of boundary conditions has the form: $\psi=\frac{\partial^2\psi}{\partial z^2}=\tau=s=0 \quad (z=0,1)$ at $z=0,1$.

Let us seek the solution of the system (1-3) by Bubnov-Galerkin method in the form satisfying boundary conditions:

$$\Psi = \sum sin(j\pi z)\Psi_j(x,t); \quad T = \sum sin(j\pi z)T_j(x,t); \quad S = \sum sin(j\pi z)S_j(x,t);$$



Periodic regime after two bifurcations of period doubling and corresponding Poincare map. $R_T = 9090$; $R_S = 8000$; $\sigma = 1$.

Figure 1 illustrates periodic regime after two bifurcations of period doubling and the corresponding Poincare map. In Figure 2 we have Poincare map for stochastic regime after the sequence of bifurcations. In [2] you can find more details on sequence of bifurcations. With the further increase of R_T the reverse process begins, the Mobius band is cut along itself (by cutting the Mobius band we again obtain the Mobius band due to non-orientability of the surface). Finally we get periodic solution and after that stochastic modes develops again to more complicated modes.

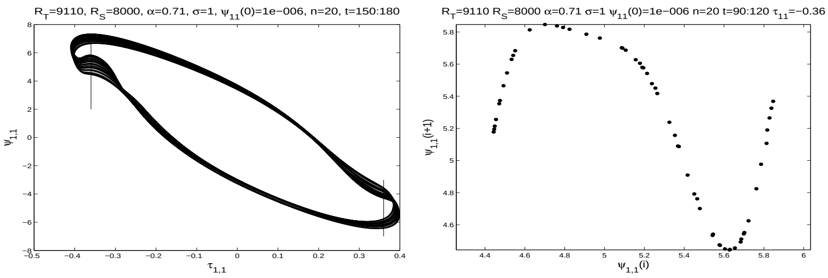


Figure 2. A projection of the attractor in the phase space on the plane of the first harmonics (ψ_{11}, τ_{11}) and the corresponding Poincare map.

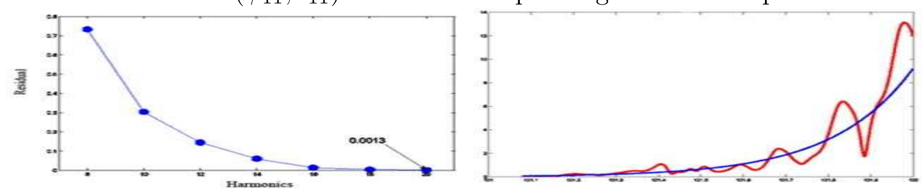


Figure 3. Dependence of the relative residual on the number of harmonics and divergence of trajectories on the attractors (Lyapunov exponent $\lambda = 5.3276$; $R_T = 15000$; $R_S = 15000$; $\sigma = 1$)

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References

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