

Precession Topological Solitons with Nonzero Hopf Invariant in a Uniaxial Ferromagnet

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1 Introduction

In models with the three-component unit vector field $\mathbf{n} = (n_1, n_2, n_3)$, where $\mathbf{n}^2 = 1$, localized structures exist if the field \mathbf{n} asymptotically approaches the ground state value $\mathbf{n}_0 = (0, 0, 1)$ as $|\mathbf{r}| \rightarrow \infty$. Such fields map the $\mathbb{R}^3 \cup \{\infty\}$ space to the two-dimensional sphere S^2 and are classified by the homotopy classes $\pi_3(S^2) = \mathbb{Z}$ and characterized by the Hopf invariant H . Stable three-dimensional solitons in uniaxial ferromagnet with $H \neq 0$ have been discussed since the pioneering works by Dzyaloshinskii and Ivanov [1]. Simple reasoning based on the Derrick theorem indicates the absence of nontrivial static three-dimensional solitons with finite energy in the aforementioned media. However, dynamical structures of this kind, which are stabilized by the precession of the magnetization, can exist [2]. Constant-velocity precession solitons with $H = 1$ were numerically found in the Heisenberg model for an isotropic ferromagnet [3]. Both stationary (magnon drops) [2] and uniformly moving [4] precession radially symmetric nontopological solitons in a uniaxial ferromagnet were analyzed. In addition, the moving topological soliton was recently analyzed [5], but stable configurations with $H \neq 0$ were not found. In this work, we numerically find stationary [6] and uniformly moving precession three-dimensional solitons with nonzero Hopf indices, in a uniaxial ferromagnet. Hereafter, fine structure of such solitons and main features are studied.

2 Stationary and moving solitons

The dynamics of the magnetization vector is described by the Landau-Lifshitz equation. In the case of negligible relaxation, this equation has the form

$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{2\mu_0}{\hbar} \left[\frac{\delta E}{\delta \mathbf{M}} \times \mathbf{M} \right], \quad (1)$$

where the ferromagnet energy

$$E = E_{exch} + E_{anis} = \frac{\alpha}{2} \int \left(\frac{\partial \mathbf{M}}{\partial r_i} \right)^2 d\mathbf{r} + \frac{\beta}{2} \int (M_x^2 + M_y^2) d\mathbf{r} \quad (2)$$

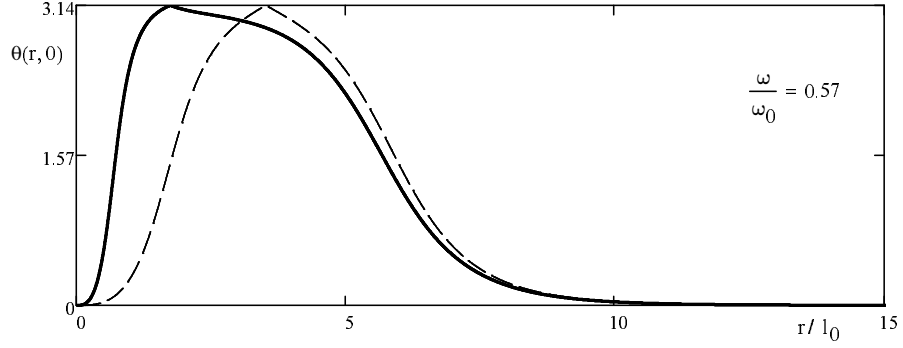


Figure 1: Shape of stationary (solid line) and moving (dashed line) with speed $V = 0.11l_0\omega_0$ soliton, for $H = 3$; here ω_0 is the FMR frequency, $l_0 = \sqrt{\alpha/\beta}$.

Using the parameterization

$$\mathbf{M} = M_0\mathbf{n}, \quad \mathbf{n} = (\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta) \quad (3)$$

we seek solutions describing precession solitons of the form

$$\Phi = \omega t + Q\varphi + \phi(r, z - Vt), \quad \Theta = \theta(r, z - Vt), \quad (4)$$

where φ is the polar angle of the cylindrical coordinate system (r, φ, z) . It is known that studying the dynamics of such structures is equivalent to solving a variational problem of minimizing the energy functional E for a fixed invariants of motion N (total number of spin deviations) and P_z (momentum). The energy with an additive penalty functions was minimized by the conjugate gradient method. Imposed constraint on the N integral is taken into account by applying a linear operator for each iteration. A quadratic penalty functions imposes a constraint on the P_z integral value and on the norm of the vector at any numerical-grid node. For the solitons presented in our work, the Hopf index $H = Q$. Analysis showed [6] that it is most difficult to simulate solitons with $H = 2$ and especially with $H = 1$. A series of efficacious numerical results were obtained for $H = 3$. Figure 1 shows the profile of stationary ($V = 0$) and moving ($V > 0$) soliton with the same precession frequency. The discovered solitary structures can potentially be used for information recording and storage in three-dimensional samples.

References

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