

# On a nonlocal Ostrovsky-Whitham type integrable dynamical system

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## 1 The Ostrovsky-Whitham medium model with spatial memory

We consider an Ostrovsky-Whitham type nonlinear medium model described by means of the evolution equation  $du/dt = 2uu_x + \int_{\mathbb{R}} \mathcal{K}(x, s)u_s ds$ , discussed first in [2]. If the corresponding Whitham kernel  $K$  looks like  $\mathcal{K}(x, s) := -\theta(x-s)u_s$  for  $x, s \in \mathbb{R}$ , naturally modeling [1] the relaxing spatial memory effects, the resulting equation becomes

$$du/dt = 2uu_x - \partial^{-1}u_x^2 := K[u], \quad (1)$$

is strongly nonlocal and appears to possess very interesting mathematical properties.

## 2 The regularization and Lax type integrability

We proved that the following regularized nonlinear dynamical system

$$\left. \begin{aligned} u_t = 2uu_x - v, \quad v_t = 2uv_x \end{aligned} \right\} := K[u, v] \quad (2)$$

of hydrodynamic type, being already well defined on the extended  $2\pi$ -periodic functional space  $\mathcal{M} := C_{2\pi}^{\infty}(\mathbb{R}; \mathbb{R})$  and completely equivalent to that given by expression (1), is Lax type integrable bi-Hamiltonian flow.

## References

- [1] Ostrovsky L.A. Nonlinear internal waves in a rotating ocean., *Oceanology*, 1978, v. 18, p.119-125.
- [2] Whitham G.B. *Linear and Nonlinear Waves*. Wiley-Interscience, New York, 1974, 221p.