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Heat operator: twisting transformations and extended resolvent

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Literature reports on solutions of the nonlinear (2+1)-dimensional equations that show interesting geometrical structures. In particular, for the KP equation there exists a class of solutions rapidly decaying in all spatial directions with exception of a finite number of rays, where they have finite (nonzero) limits. Characteristic property of the KPII equation is that in a generic situation numbers of "ingoing" and "outgoing" rays are different and they are not obliged to be parallel. In addition, these solutions, during their time evolution, display spatial interaction patterns, resonances and web structures.

A rich class of such solutions is given by N-soliton ones being perturbed by an arbitrary smooth and rapidly decaying function of the both spatial variables. The goal of investigation of these solutions can be achieved by looking not directly to the solutions in the space and to their evolution, but to their spectral data, that must be obtained by extending the Inverse Scattering Theory to such potentials with non-decaying asymptotic behavior. These spectral data are expected to have a complicated geometrical structure, but, since KPII is integrable, a trivial time evolution.

We present results of the study of the spectral properties of the heat operator which is the linear spectral problem associated to KPII. To this end we use the extended resolvent approach, that has been developed to extend the inverse spectral transform for (2+1)-dimensional equations to a general class of potentials, including potentials that are not decaying along a finite number of directions in the plane. We introduce twisting transformations of the heat operator are introduced and use them, at the same time, to superimpose à la Darboux N solitons to a generic smooth, decaying at infinity, potential and to generate the corresponding Jost solutions. These twisting operators are also used to study the existence of the related extended resolvent. Existence and uniqueness of the extended resolvent is considered in detail by means of the special examples. In particular, we show that here there exist annihilators of the extended heat operator that are exponentially decaying functions of spatial variables. Existence of these annihilators is the new and unexpected feature of such potentials, that has no analog either in the one-dimensional, or in a one-soliton two-dimensional cases.

Initial results of the study are presented in [1].

References

[1] M. Boiti, F. Pempinelli, A.K. Pogrebkov, B. Prinari, "Building extended resolvent of the heat operator via twisting transformation", arXiv:0901.3857v1 [nlin.SI] (to appear in Theor. and Math. Phys. (2009))