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Loop coproducts

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The coproduct method is a method for the construction of classical and quantum integrable systems. It allows to build up from a Poisson-Hopf algebra (a non-commutative Hopf algebra) a set of involutive functions (commuting operators) on the N-th tensor product of the Hopf algebra, with N an arbitrary integer. Since quantum algebras are instances of Hopf algebras and their Poisson analogues are instances of Poisson-Hopf algebras, quantum algebras are a natural object of interest for the coproduct method. Coproduct method has been successfully employed to construct and eventually solve classical and quantum integrable systems with a finite, but arbitrary, number of degrees of freedom of physical interest, both on flat and curved spaces with radial symmetry [1, 2, 3, 4]. One of the limitations of the coproduct method is that, in some cases, for example when it is applied to the standard Hopf algebra structure of a simple Lie-Poisson algebra of rank greater than one, the number of functions in involution is not sufficient to grant complete integrability. We show that, in the classical case, it is possible to generalise the coproduct method by weakening the isomorphism request on the coproduct map. Through this generalisation it is possible to recover complete integrability for simple Lie-Poisson algebras of arbitrary rank.

References

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