NEEDS 2009 Workshop
Nonlinear Evolution Equations and Dynamical Systems
Isola Rossa, May 16–23, 2009.

Universality in the two matrix model

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March 23, 2009

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The two-matrix model is a generalization of the one matrix model defined by the probability measure

$$Z_n^{-1} \exp\left(-n\left(\mathrm{tr}(V(M_1) + W(M_2) - \tau M_1 M_2\right)\right) \mathrm{d}M_1 \mathrm{d}M_2,$$

on the space of pairs (M_1, M_2) of $n \times n$ Hermitian matrix.

One of the major problems is to obtain asymptotics for the eigenvalue statistics as $n \to \infty$, which is encoded in a correlation kernel that can be expressed in terms of biorthogonal polynomials. Until recently, rigorous results in this direction is limited to the case when either V or W in the measure is a quadratic polynomial. Due to recent advancement in the generalization of the Deift-Zhou steepest descend method to multi-dimensional Riemann-Hilbert problems, Duits and Kuijlaars [1] were able to obtain rigorous asymptotic results for the 2 matrix model with more general potentials V and W for the first time. In the large n limit, the eigenvalues of the matrix M_1 is supported on a finite union of intervals on the real axis. In the work of Duits and Kuijlaars [1], results were obtained for the case when the eigenvalues of M_1 is supported on a single interval in the asymptotic limit. Generalization of these results to the case when eigenvalues are supported on multi cuts requires the use of Riemann theta functions [2]. In this talk I will discuss some ideas behind these recent results and their relation with Riemann surfaces and theta functions.

References

[1] M. Duits and A. B. J. Kuijlaars. Universality in the two matrix model: a Riemann-Hilbert steepest descent analysis. To appear in *Comm. Pure Appl. Math. arXiv:0807.4814.*

[2] M. Y. Mo. Universality in the two matrix model with a monomial quartic and a general even polynomial potential. *arXiv:0811.0620.*