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The universal laws of Random Matrices

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1 Summary

Eigenvalues of random matrices display universal behavior in two ways. On the one hand, local eigenvalue statistics depend for large matrix dimensions only on the symmetries of the matrices but not on the details of the chosen probability measure. On the other hand, these distributions appear in a number of seemingly unrelated combinatorial models and even in number theory.

In this talk we will mainly focus on the first aspect of universality. We will briefly review what is known about the eigenvalue statistics for various types of random matrices with a special focus on orthogonal and symplectic ensembles ($\beta = 1,\ 4$) that are invariant under suitable changes of bases. For these ensembles we will sketch the proofs that are all based on the method of orthogonal polynomials as described in [6, 5]. In [1, 2] the proof of universality is then given by a detailed analysis of the asymptotic behavior of the orthogonal polynomials. The somewhat different approaches of [3, 4] will also be discussed.

References

[1] Deift, P. and Gioev, D.: Universality in random matrix theory for orthogonal and symplectic ensembles. *Int. Math. Res. Papers* **2007** (2007) 4–116.

[2] Deift, P., Gioev, D., Kriecherbauer, T. and Vanlessen, M.: Universality for orthogonal and symplectic Laguerre-type ensembles. *J. Stat. Phys.* **129** (2007) no. 5-6, 949–1053.

[3] Shcherbina M.: On Universality for Orthogonal Ensembles of Random Matrices. *Preprint* arXiv:math-ph/0701046.

[4] Stojanovic, A.: Universality in orthogonal and symplectic invariant matrix models with quatric potentials. *Math.Phys.Anal.Geom.* **3** (2002) 339–373.

[5] Widom, H.: On the relation between orthogonal, symplectic and unitary matrix ensembles. *J. Stat. Phys.* **94** (1999) no. 3-4, 347–363.

[6] Tracy C.A. and Widom, H.: Correlation functions, cluster functions, and spacing distributions for random matrices. *J. Stat. Phys.* **92** (1998) 809–835.