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On representations of two-dimensional surfaces obtained from CP^{N-1} sigma models

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The purpose of this talk is to construct two-dimensional surfaces immersed in multidimensional Euclidean spaces using the Weierstrass technique for harmonic maps $S^2 \to CP^{N-1}$. The subject was studied by several authors (see e.g. [1, 2, 3, 4]) who produced several variants of the Weierstrass representation. For a comprehensive review of this topic see e.g. [5, 6, 7, 8] and references therein.

It will be demonstrated that if the CP^{N-1} model equations are defined on the S^2 sphere and the associated action functional is finite, then the generalized Weierstrass formula for immersion describes conformally parametrized surfaces immersed in the $su(N)$ algebra [9]. In particular, for any holomorphic or anti-holomorphic solution of the model, the associated surfaces can be expressed in terms of an orthogonal projector of rank $N-1$. The realization of the method is demonstrated for the two-dimensional conformally parametrized surfaces immersed in the $su(3)$ algebra. The approach is illustrated through examples, including dilation-invariant meron-type solutions and the Veronese solutions for the CP^2 model. In addition, it is shown that the two-dimensional surfaces obtained from the generalized Weierstrass formula coincide with the ones obtained from the Sym-Tafel formula [10, 11, 12, 13]. These two approaches correspond to the parametrizations of one and the same surface immersed in the $su(N)$ algebra.

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