NEEDS 2009 Workshop
Nonlinear Evolution Equations and Dynamical Systems
Isola Rossa, May 16–23, 2009.

Quantum random walks and orthogonal polynomials

F. Alberto Grünbaum[a]

February 26, 2009

a. Math Dept, UC Berkeley.

Abstract

This is joint work with M.J. Cantero, L. Moral and L. Velazquez from Zaragoza, Spain.

We consider quantum random walks (QRW) on the integers, a subject that has been considered in the last few years in the framework of quantum computation.

We show how the theory of CMV matrices gives a natural tool to study these processes and to give results that are analogous to those that Karlin and McGregor developed to study (classical) birth-and-death processes using orthogonal polynomials on the real line.

In perfect analogy with the classical case the study of QRWs on the set of non-negative integers can be handled using scalar valued (Laurent) polynomials and a scalar valued measure on the circle. In the case of classical or quantum random walks on the integers one needs to allow for matrix valued versions of these notions.

We show how our tools yield results in the well known case of the Hadamard walk, but we go beyond this translation invariant model to analyze examples that are hard to analyze using other methods. More precisely we consider QRWs on the set of non-negative integers. The analysis of these cases leads to phenomena that are absent in the case of QRWs on the integers even if one restricts oneself to a constant coin. This is illustrated here by studying recurrence properties of the walk, but the same method can be used for other purposes.