

Singular Finite-Gap Operators and Indefinite Metric

Petr Grinevich^a, Sergei Novikov^{a,b}

March 1, 2009

- a. L.D.Landau Institute for Theoretical Physics of RAS, ul. Ak. Semenova 1a, Chernogolovka, Moscow Region, 142432, Russia.
- b. Institute for Physical Science and Technology, University of Maryland at College Park, College Park, 20742, MD, USA

Many "real" inverse spectral data for periodic finite-gap operators (consisting of Riemann Surfaces with marked "infinite points" and Divisors) lead to operators with real but singular coefficients. These operators cannot be considered as self-adjoint in the ordinary (positive) Hilbert spaces of functions of x . In particular, it is true for the special case of the Lamé' operators with elliptic potential where eigenfunctions were found in XIX Century by Hermit. However, exactly these Baker-Akhiezer functions serve according to the ideas of the works [1, 2] as a right analog of the Fourier Transform on Riemann Surfaces. It turns out that these operators for genus one and more are symmetric in some indefinite inner product, described in this work. The analog of Fourier Transform is an isometry in this inner product.

References

- [1] I.M.Krichever, S.P.Novikov: Algebras of Virasoro type, Algebras of virasoro type, riemann surfaces and structures of the theory of solitons *Funktsional. Anal. Appl* **21** (1987), 126–142.
- [2] P.G.Grinevich, S.P.Novikov: Topological charge of the real periodic finite-gap Sine-Gordon solutions. *Communications on Pure and Applied Mathematics* **56:7** (2003) 956–978.