

Systoles and geodesic length spectrum on compact Riemann surfaces

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1 Extended Abstract

We establish systolic inequalities, compute precise values for systolic quantities and study its behavior with the variation of the Fenchel-Nielsen coordinates on a compact Riemann surface of genus 2.

The metric and geometric structure from the surfaces can be studied by the closed geodesics spectrum and the Laplace-Beltrami operator spectrum. To obtain these spectra it is not easy but more difficult is to describe their dependence with the parameters which determine the metric and geometric structure of the surface. We study these spectra dependence through the boundary map when a Riemann surface M of genus 2, thus with negative curvature, is considering. From a classical point of view the hyperelliptic surfaces are the most simple Riemann surfaces ([5]).

The systole of a compact Riemann surface is defined like the minimal length of a noncontractile curve. The systole of surfaces of Riemann was prone in-depth studies during the decade 1993-2003. The articles of P. Schmutz Schaller marked an important projection in the search for maximum surfaces for the systole ([7], [8]). That of C Bavard ([1]), provides as-with him a pleasant theoretical framework, copied on that of the network analysis. The present paper is part of a program to understand the behavior of the spectrum of the Laplacian of a compact Riemannian manifold M and geodesic length spectrum of M , endowed with a metric of constant curvature -1 .

We use techniques link between combinatorial structures (symbolic dynamics) and algebraic-geometric structures. Given a compact Riemannian manifold M , the Laplace-Beltrami operator is an elliptic operator with discrete spectrum. For surfaces of genus 2 we computed the geodesic length spectrum of M (lengths of closed geodesics) ([3], [4]). These surfaces was obtained by gluing together pairs of pants with no twists on the boundary components. This corresponds to Riemann surfaces in Teichmüller space ([2], [6]).

The set of equivalence classes of hyperbolic metrics (or equivalently complex structures) under orientation preserving diffeomorphisms on M forms the moduli space \mathcal{M} of compact Riemann surfaces of genus t . It is represented by a quotient

space $M=H^2/G$ of the upper half-plane H^2 by a Fuchsian group G which is isomorphic to the fundamental group of M . The discrete group G is identified with the corresponding system of generators. A fundamental domain F is defined. The method is to decompose Riemann surface into a set of 2 pairs of pants by simple closed geodesics. Then the Fenchel-Nielsen coordinates are defined by geodesic length functions of three simple closed geodesics. With explicit constructions and side pairing transformations ([4]), we define the Fuchsian group G representing the closed Riemann surface of genus 2.

The paper is organized as follows. In Sec. II we briefly introduce a geometric description of the surface, the universal covering space, the Fenchel-Nielsen parameters, that are a more familiar way of parameterizing Teichmüller space, and a explicit geometrical construction of the fundamental domain. In Sec III we detail the construction of the boundary map, the Markov matrix associated to and compute the generators of the fundamental group. Finally, in Sec IV we establish systolic inequalities, compute precise values for systolic quantities and study its behavior with the variation of the Fenchel-Nielsen coordinates

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