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# Interaction length of DM solitons in the presence of third order dispersion with loss and amplification

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Dispersion Management (DM) techniques have become a fundamental technology for high-speed soliton transmission in long-haul optical communication links, allowing to reduce several system impairments such as four-wave mixing and Gordon-Haus timing jitter while improving the signal-to-noise ratio [1]. One of the most important penalties associated with propagation in a single frequency channel arises from the interaction of neighboring pulses [2]. Besides, as transmission bit rates move to higher standards of the synchronous optical network or the synchronous digital hierarchy, the impact of third-order dispersion (TOD) effects becomes increasingly important, even for a single channel, due to the reduction of pulse width.

We study the effect of TOD on the interactions of DM solitons in the presence of loss and amplification. The analysis is based in an ODE model obtained by means of a variational method [3] which takes into account third order dispersion [4]. This permits to reduce the full complexity of the generalized Nonlinear Schrödinger Equation (GNLSE) that models the propagation of pulses in an optical fiber in the presence of TOD [4]. The system dynamics are studied by means of a set of ODEs which capture the most relevant features of the evolving solutions in an approximate manner. Assuming a two-pulse Gaussian ansatz in the GNLSE, one obtains the equations of motion [4]

$$\frac{dp_l}{dZ} = -C_l p_l^3 \left(D - 6\delta\omega_l\right) \tag{1}$$

$$\frac{dC_l}{dZ} = \left(1 + C_l^2\right)\left(D - 6\delta\omega_l\right) p_l^2 - \frac{p_l}{S(Z)}\sqrt{2\pi}\left(E_l + 2E_{3-l}p_l^3\left(1 - \tau_l^2\right)\exp\left(-\frac{\tau_l^2}{2}\right)\right) \tag{2}$$

$$\frac{dT_l}{dZ} = -D\omega_l + 3\delta\omega_l^2 + \frac{3}{2}\delta\left(1 + C_l^2\right)p_l^2 \tag{3}$$

$$\frac{d\omega_l}{dZ} = \frac{2E_{3-l}}{S(Z)}\sqrt{2\pi}\tau_l P^2 \exp\left(-\frac{\tau_l^2}{2}\right), \tag{4}$$

where $l = 1, 2$, and $p$, $C$, $T$, $\omega$ and $E$ are, respectively, the inverse pulse width, chirp parameter, pulse position, frequency shift and energy. Parameter $\delta$ accounts for TOD effects.

The dynamics of single pulse solutions are adequately described in the $(p, C)$ phase plane. For a lossless system, the trajectories drawn by the system evolution remain symmetric relative to the line $C(Z) = 0$ [4]. We also analyze the energy and initial chirp required to obtain stable periodic pulse propagation in the presence of loss and amplification. In this case, the system dynamical path in phase space is not symmetric with respect to the $C(Z) = 0$ axis and the chirp-free point is no longer at the midpoint of the anomalous dispersion segment.

The simultaneous presence of two pulses produces a shift of their center frequencies, caused by XPM, and a corresponding displacement of their center positions due to group velocity dispersion in the transmission medium. In the lossy case, with amplifiers periodically placed at fixed positions in the dispersion map, there is a net increase in the interaction distance as we place an amplifier at specific positions of the dispersion map period for some values of the dispersion difference. In this work, we extend previous analyzes of the effect of the TOD on the interaction length [5] to the case where loss and amplification are present in the transmission link. We find that TOD can substantially reduce the interaction distance.

Finally, the results obtained from the integration of the ODE system are compared with direct solutions of the full GNLSE using the split-step Fourier method. Excellent agreement is found between the two sets of values for all the cases studied.

# References

[1] L.F. Mollenauer and J.P. Gordon, Solitons in optical fibers: fundamentals and applications. *Elsevier/Academic Press* (2006).

[2] T. Inoue, H. Sugahara, A. Maruta and Y. Kodama: Interactions Between Dispersion Managed Solitons in Optical-Time-Division-Multiplexed Systems. *IEEE Photon. Technol. Lett.* **12** (2000) 299.

[3] D. Anderson: Variational approach to nonlinear pulse propagation in optical fibers. *Phys. Rev. A* **27** (1983) 3135.

[4] F. J. Diaz-Otero and P. Chamorro-Posada: Interchannel soliton collisions in periodic dispersion maps in the presence of third order dispersion. XVII International Conference on Nonlinear Evolution Equations and Dynamical Systems (NEEDS 2007). *J. Nonlinear Math. Phys.*, **15** Supp.3 (2008) 137–143.

[5] F. J. Diaz-Otero, P. Chamorro-Posada and J.C. Garcia-Escartin: Dispersion-managed soliton interactions in the presence of third-order dispersion. II International Conference on Advanced Optoelectronics and Lasers (CAOL 2005). *CAOL Proceedings*, (2005) 153–155.