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# Isochronous dynamical systems and the arrow of time

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A vector-valued time-dependent function is called *isochronous* if all its components are periodic in time with the same fixed period $T$. A dynamical system is called *isochronous* if its *generic* solution is *isochronous*: periodic in all its degrees of freedom with a fixed period $T$ independent of the initial data. It will be shown how essentially any (autonomous) dynamical system can be modified into another (also autonomous) dynamical systems which is *isochronous* with an (arbitrarily!) assigned period $T$, and which moreover behaves, over time periods very short with respect to $T$, essentially as the original (unmodified) system – up to a constant time rescaling. This can also be done for a large class of Hamiltonian systems (both the unmodified and the modified one), including the Hamiltonian describing the most general many-body problem (provided it is, overall, translation-invariant). Some implications of this fact for statistical mechanics and thermodynamics will be mentioned, and for the distinction among integrable and nonintegrable dynamical systems (all isochronous systems are integrable, in fact maximally superintegrable). These findings have all been obtained together with F. Leyvraz: some of them are reported in my monograph [1] entitled *Isochronous systems* (Oxford University Press, 2008), others are more recent, [2]-[11].

## References

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