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Impact of Higher-Order Fibre Dispersion on the Evolution of Parabolic Optical Pulses

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1 Introduction

Recent developments in nonlinear optics reveal an interesting class of pulses with a parabolic intensity profile in the energy-containing core and a linear frequency chirp that can propagate in a fibre with normal second-order (group-velocity) dispersion (SOD) – see, e.g., [?] and references therein. Parabolic pulses propagate in a stable self-similar manner, holding certain relations (scaling) between pulse power, width, and chirp parameter. In the additional presence of linear amplification, they enjoy the remarkable property of representing a common asymptotic state (or attractor) for arbitrary initial conditions. Analytically, self-similar (SS) parabolic pulses can be found as asymptotic, approximate solutions of the nonlinear Schrdinger equation (NLSE) with gain in the semi-classical (large-amplitude/small-dispersion) limit. By analogy with the well-known stable dynamics of solitary waves – solitons, these SS parabolic pulses have come to be known as similaritons.

In practical fibre systems, inherent third-order dispersion (TOD) in the fibre always introduces a certain degree of asymmetry in the structure of the propagating pulse, eventually leading to pulse break-up. To date, there is no analytic theory of parabolic pulses under the action of TOD. In this work, we develop a perturbation analysis that describe the effect of weak TOD (and/or higher-order dispersion) on the parabolic pulse solution of the NLSE in a fibre gain medium.

2 Perturbation theory

Seeking a solution of the NLSE propagation model of the form $u(z,t) = a(z)F(\eta,\xi)\exp[iC(z)t^2]$, where $\xi = t/\tau(z)$ and $\mathrm{d}\eta/\mathrm{d}z = \sigma a^2(z)$, yields coupled evolution equations for the pulse peak amplitude a, characteristic width τ and chirp parameter C, along with a governing equation for the structural function F [?]. Here we consider the case of a constant gain profile along the fibre: $g(z) = g_a$. We observe that there is a natural small parameter in this problem: $\epsilon \equiv \tau(z)/a(z) = \tau(z_0)/a(z_0) = (3/2)\sqrt{\sigma\beta_2/(2g_a^2)} \ll 1$

($\epsilon \sim 0.03$ for typical SOD coefficient $\beta_2 = 35 \times 10^{-3}\,\mathrm{ps}^2/\mathrm{m}$, Kerr nonlinearity coefficient $\sigma = 6 \times 10^{-3}\,(\mathrm{W\,m})^{-1}$ and $g_a = 1.44\,\mathrm{m}^{-1}$). The assumption $|\eta - \eta_0| \ll 3\sigma\tau_0^2/(2\epsilon^2 g_a)$ allows us to remove the dependence on η of the coefficients of the resulting evolution equation for F. This condition, however, imposes a limit on the propagation distances where our model holds. For instance, for the fibre parameters mentioned above, the available propagation distances are limited to $z \sim 0.5\,\mathrm{m}$ or $\eta \sim 30$. Extension of the propagation distance can be obtained by decreasing g_a. Making use of the small parameter ϵ, we expand F as $F(\eta,\xi) = [A_0(\eta,\xi) + \epsilon A_1(\eta,\xi) + \ldots]\exp\{i[\phi_0(\eta,\xi) + \epsilon\phi_1(\eta,\xi) + \ldots]\}$. After application of standard perturbation theory, we find a self-similar solution with a parabolic distribution of the intensity and trivial phase [**?**]: $A_0(\eta,\xi) = \sqrt{\lambda(1-\xi^2)}$, $\phi_0(\eta,\xi) = \lambda\eta$, at $O(\epsilon^0)$. The leading order perturbation comes at $O(\epsilon^2)$, where we obtain the solution

$$A_2(\eta,\xi) = R(\xi)(\eta-\eta_0), \quad \phi_2(\eta,\xi) = R(\xi)A_0(\xi)(\eta-\eta_0)^2 + G(\xi)(\eta-\eta_0),$$

$$R(\xi) = \frac{\beta_3}{6\sigma}\left[\frac{1}{\tau_0^5}A_{0\xi\xi\xi} - \frac{12C^2}{\tau_0}\xi(\xi A_0)_\xi\right],$$

$$G(\xi) = -\frac{\beta_2}{2\sigma\tau_0^4}\frac{A_{0\xi\xi}}{A_0} + \frac{\beta_3}{6\sigma}\left[\frac{6C}{\tau_0^3}\frac{(\xi A_{0\xi})_\xi}{A_0} - 8C^3\tau_0\xi^3\right] \qquad (1)$$

In (**??**), β_3 is the TOD coefficient ($|\beta_3| \ll |\beta_2|$). The perturbation in (**??**) is singular at the points where the purely parabolic solution A_0 tends to zero. To remove this problem, here we use a particular approximation for A_0 [**?**], which coincides with the parabolic pulse shape in the central core and, at the same time, approximates the smooth decrease of the amplitude in the pulse tails which is observed on the true asymptotic pulse solution.

The theoretical model developed here is capable of predicting with sufficient accuracy the pulse structural changes induced by TOD, which are observed through direct NLSE numerical simulations. This perturbation analysis can be extended also to include the effect of higher-order dispersion terms.

3 Conclusion

We have presented a perturbation approach to describe the structural changes induced by TOD on the parabolic pulse solution of the NLSE in a fibre gain medium. The theoretical model fairly reproduces the results of direct NLSE numerical simulations. Further details of the model and comparison with simulations, as well as inclusion of higher-order dispersion terms and connection with practical applications will be presented in the conference.

References

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