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Extended resolvent approach to Inverse Scattering in multidimensions and its applications

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The Inverse Scattering Transform method, in the last three decades, was applied to a very broad class of differential and discrete nonlinear equations in 1+1 and 2+1 dimensions. As regards the integrable equations in 2+1 dimensions, the theory was initially developed considering solutions that are decaying to zero at large distances in the plane. However, integrable multidimensional evolution equations incorporate, in general, solutions of their (1+1)-dimensional reductions and serious difficulties have been found in order to extend the theory to bidimensional solutions that have wave behavior at large distances.

We developed the method of the Extended Resolvent, which resulted to be an effective method for the investigation of multidimensional scattering problems. On this base the class of potentials of such problems that admit investigation by means of the Inverse Scattering theory was essentially extended. In particular, potentials nondecaying at space asymptotics were included and the method was successfully applied to the case of solutions of KPI describing N solitons on a generic background.

However, serious and unexpected additional difficulties have been found in the case of KPII. Some preliminary results have been already obtained, but the work is in progress and A.K. Pogrebkov programs to give another talk at the Workshop on these preliminary results and on what additionally we hope to get in the meantime, with special attention to the geometrical properties of singularities of the related Extended Resolvent in the case of N soliton solutions.

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