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Non-Hamiltonian generalizations of dispersionless 2DTL hierarchy

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$$
\begin{aligned}
u_{xt} &= u_{yy} + (uu_x)_x + v_x u_{xy} - u_{xx} v_y, \\
v_{xt} &= v_{yy} + u v_{xx} + v_x v_{xy} - v_{xx} v_y,
\end{aligned} \tag{1}
$$

having a Lax pair

$$
\begin{aligned}
&\partial_y \mathbf{\Psi} = ((\lambda - v_x)\partial_x - u_x \partial_\lambda)\mathbf{\Psi}, \\
&\partial_t \mathbf{\Psi} = ((\lambda^2 - v_x\lambda + u - v_y)\partial_x - (u_x\lambda + u_y)\partial_\lambda)\mathbf{\Psi}.
\end{aligned}
$$

For $v = 0$ the system (1) reduces to the dKP (Khohlov-Zabolotskaya) equation

$$
u_{xt} = u_{yy} + (uu_x)_x. \tag{2}
$$

Respectively, $u = 0$ reduction gives an equation [3]

$$
v_{xt} = v_{yy} + v_x v_{xy} - v_{xx} v_y. \tag{3}
$$

The hierarchy related to this system was studied in [4, 5]. We consider generalizations of dispersionless 2DTL hierarchy connected with non-Hamiltonian vector fields. They form one-parametric family with reciprocal transformations acting as equivalence relations. A simplest generalization of dispersionless 2DTL equation reads

$$
\begin{aligned}
&(\mathrm{e}^{-\phi})_{tt} = m_t \phi_{xy} - m_x \phi_{ty}, \\
&m_{tt}\mathrm{e}^{-\phi} = m_{ty} m_x - m_{xy} m_t,
\end{aligned} \tag{4}
$$

with a Lax pair

$$
\begin{aligned}
&\partial_x \mathbf{\Psi} = \left((\lambda + \frac{m_x}{m_t})\partial_t - \lambda(\phi_t \frac{m_x}{m_t} - \phi_x)\partial_\lambda \right)\mathbf{\Psi}, \\
&\partial_y \mathbf{\Psi} = \left(\frac{1}{\lambda}\frac{\mathrm{e}^{-\phi}}{m_t}\partial_t + \frac{(\mathrm{e}^{-\phi})_t}{m_t}\partial_\lambda \right)\mathbf{\Psi}
\end{aligned}
$$

For $m = t$ system (4) reduces to dispersionless 2DTL equation

$$(\mathrm{e}^{-\phi})_{tt} = \phi_{xy},$$

Respectively, $\phi = 0$ reduction gives an equation [3]

$$m_{tt} = m_{ty} m_x - m_{xy} m_t.$$

System (4) doesn't preserve the symmetry of dispersionless 2DTL equation with respect to x, y variables, however, we also introduce symmetric generalization of d2DTL equation. Generating relations for the hierarchy and Lax-Sato equations are introduced. The dressing scheme based on vector nonlinear Riemann problem

$$\mathbf{\Psi}_+ = \mathbf{F}(\mathbf{\Psi}_-), \tag{5}$$

and some reductions are discussed.

References

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