C-integrable systems of discrete lattices.

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We examine system of fully discrete "hyperbolic" equations

$$u_{i+1,j+1} = f(u_{i+1,j}, u_{i,j+1}, u_{i,j}),$$

where i, j are integer indices, u_{ij} is a vector of n unknown sequences, f is a vector of n known functions. The problem is to find conditions for so-called C-integrability, or Darboux integrability which means existance of sufficient number of i- and j-integrals for initial system. The integrals are $I(u)$ such that $(T_i - 1)I(u) = 0$, (T_i is shift in i) and the same for J: $(T_j - 1)J(u) = 0$. Conditions of C-integrability for linear systems were written out explicitly in the form of proven statements and were illustrated by appropriate examples. For the systems equality of generalized Laplace invariant to zero is the condition of integrability. System is called "C-integrable" if its lattice of generalized Laplace invariants breaks at both directions. Further, for integrable systems the Laplace procedure was applied for obtaining generic solutions. For nonlinear systems the concept of C-integrability should be understood as C-integrability for appropriate "linearized" equation. There is a procedure for constructing complete set of integrals for those systems.