

# New Generalized Solutions to Systems of Nonlinear PDEs

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A new type of generalized solution to systems of nonlinear PDEs is constructed as the uniform convergence space completion of spaces of piecewise differentiable functions. These generalized solutions have the distinct advantage of being actual functions on the domain  $\Omega \subseteq \mathbb{R}^n$  of the PDEs, as apposed to other types of generalized solutions, like distributions, which are usually considered. In fact, they are Hausdorff continuous functions on  $\Omega$ , see [1]. The rudiments of a differential calculus for Hausdorff continuous functions, which extends the classical theory for real valued functions, is presented. The resulting extended differential operators are used, in conjunction with the natural order on the spaces of functions, to define suitable uniform convergence structures in the sense of [3], see for instance [2], [16] and [17]. As an illustration of the general approach applying to arbitrary continuous nonlinear systems of PDEs, we obtain existence of generalized solutions to the Navier-Stokes equations.

## References

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