Peakons and Biorthogonal Polynomials

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The peakon solutions to the Degasperis-Procesi (DP) equation were constructed by H. Lundmark and J. S. [1]. The main part of the construction has to do with a generalization of an inhomogenous string problem studied in the 50s by M.G. Krein to a third order analog, called a cubic string.

The peakon problem leads to an inverse problem for the cubic string which was solved completely in the cited paper. The solution introduces certain class of polynomials which share many properties with ordinary orthogonal polynomials. Yet, this new class of polynomials properly belongs to a theory of biorthogonal polynomials attached to totally positive kernels. The emerging theory of such biorthogonal polynomials is a topic of forthcoming papers by M. Bertola (Concordia), M. Gekhtman (Notre Dame) and J. S. and this talk describes a motivation behind some of these ideas viewed from the perspective of DP peakons and the cubic string.

References

[1] H. Lundmark and J. Szmigielski, Degasperis-Procesi peakons and the discrete cubic string, IMRP Int. Math. Res. Pap., **2**(53-116), 2005.