Where do braid statistics and discrete motion meet each other?

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We consider limitations of dynamics and statistics in the systems that are characterized by phase states with macroscopically large degree of degeneracy of the ground state. One of challenging phase states of this type nowadays is condensate in the form of the string-net [1]. A possible topological order [2] in such system is described by discrete equations for the set of variables containing generalized $6j$-symbols [3] and quantum dimensions $[a+1]_q$ of the Hilbert space. The topologically ordered string-net states are defined also by the string type k which has a meaning of the level in the Wess-Zumino-Witten-Novikov model or the coefficient in the $SU(N)_k$ Chern-Simons action. We focus on the discrete equations of motion which take place in the case of generalized exclusion principle statistics. In continuous limit they provide the soliton solutions of the integrable models. *We argue that in the considered case the simplest local solutions of these thermodynamic Bethe ansatz equations have the form of quantum dimension of irreducible representations of the quantum group $SU(2)_k$ like the solutions [4] in the multichannel Kondo problem.* These solutions provide an example of the point where the generalized exclusion principle statistics and braiding statistics meet each other. The general solution of the problem from the considered viewpoint could shed additional light on universal quantum computations [5] based on implication of the non-Abelian $SU(2)_2$ and $SU(2)_3$

anyons [6].

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