On differential operators on sequence spaces

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Two differential operators T_1 and T_2 on a space Λ are said to be equivalent if there is an isomorphism S from Λ onto Λ such that

$$S\,T_1 = T_2\,S.$$

The notion was first introduced by Delsarte in 1938 [?] where T_1 and T_2 are differential operators of second order and Λ a space of functions of one variable defined for $x \geq 0$. From them on several authors studied generalizations, applications and related problems [?], [?], [?], [?], [?].

In 1957 Delsarte and Lions [?] proved that if T_1 and T_2 are differential operators of the same order without singularities on the complex plane, Λ being the space of entire functions, then they are equivalent. Using sequence spaces and the fact that the space of entire functions is isomorphic to a power series space of infinite type, we find the same result in a simpler way in our opinion. The same method gives the result for differential operators of the same order with analytic coefficients on the space of holomorphic function on a disc, considering that all spaces of holomorphic functions on a disc are isomorphic to a finite power series space. The method can be applied as well to linear differential operators of the same order on other sequence spaces, finding conditions for them to be equivalent. Finally using the fact that the space $C^\infty_{2\pi}(\mathbb{R})$ of all 2π-periodic C^∞-functions on $\mathbb{R}$ is isomorphic to s, the space of rapidly decreasing sequences, we prove that two linear differential operators of order one with constant coefficients are not equivalent; this result can be extended to linear differential operators of greater order but the proof is essentially the same.

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