

# On the Hamiltonian structure of Hirota's discretization of the Euler top

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This talk deals with a remarkable integrable discretization of the  $SO(3)$  Euler top introduced in [1] by R. Hirota and K. Kimura.

A class of *implicit* discretizations of the Euler top sharing the integrals of motion with the continuous system has been presented and studied in [2].

The Hirota-Kimura discretization of the Euler top leads to an *explicit* map. Its integrability is proven by finding two independent integrals of motion and a solution in terms of elliptic functions [1]. Nevertheless the Hamiltonian formulation has not been mentioned by them. We shall give a simplified and streamlined presentation of their results and we provide a bi-Hamiltonian structure for this discretization, thus proving its integrability in the standard Liouville-Arnold sense.

The Hirota-Kimura discretization of the Lagrange top [3], as well as some preliminary results on other bilinear systems of classical mechanics, indicate the existence of a huge collection of discrete-time integrable models.

## References

- [1] Hirota, R. and Kimura, K., Discretization of the Euler top, *Journ. Phys. Soc. Jap.*, **69** (627–630), 2000.
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- [3] Hirota, R. and Kimura, K., Discretization of the Lagrange top, *Journ. Phys. Soc. Jap.*, **69** (3193–3199), 2000.