Stationary reductions of Camassa Holm equation and Universal Sato Grassmannian

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The Kadomtsev–Petviashvilii equation (KP) and its reductions can be interpreted as linear flows on the universal Sato Grassmannian $\mathcal{S}$ [5]. Moreover, these flows identify an orbit given by the action of the group GL_∞ on a suitable Fock space [4]. This second property implies the existence of a tau structure for KP and the bilinearization of every equation of the hierarchy.
However not all the integrable equations admit tau structure in the Dubrovin framework [2] and the study of such kind of systems can be useful in order to understand the relations among the three properties cited above. The Camassa–Holm hierarchy (CH) is a prototypical example of a system which does not display tau structure.
In the talk we explain the relation between CH and a linear flow on a suitable Birkoff stratum of $\mathcal{S}$. Following the geometric-Hamiltonian approach to the problem [3, 1], we identify the Noether currents of the CH system with the elements of the algebraic basis [5] of any point of $\mathcal{S}$. This construction generates a 2–component extension of the CH hierarchy which admits the local symmetries of CH as further reduction. The previous result allows us to study the stationary reductions of the CH hierarchy by means of a two step reduction process which combines a restriction on a fixed value of the Noether currents and a projection along a time of the hierarchy.
This is a joint work with G. Falqui (Milano-Bicocca).

References

[1] P. Casati, P. Lorenzoni, G. Ortenzi, M. Pedroni, *On the local and nonlocal Camassa-Holm hierarchies*, J. Math. Phys. 46 (2005), no. 4, 042704, 8 pp.

[2] B. Dubrovin, G. Zhang *Normal forms of hierarchies of integrable PDEs, Frobenius manifolds and Gromov-Witten invariants* math.DG/0108160

[3] G. Falqui, F. Magri, M. Pedroni, *Bihamiltonian geometry, Darboux coverings and Linearization of the KP hierarchy.* Commun. Math. Phys. **197** (1998), 303–324.

[4] V. G. Kac, *Infinite dimensional Lie algebras* (third edition) Cambridge University press, Cambridge, (1990)

[5] Segal, G. Wilson, G. *Loop Groups and equations of the KdV type* Publ. Math. IHES, **61**, 5–65 (1985)