Resonant excitation of multi-periodic nonlinear waves

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The nonlinear n-periodic waves are common to integrable models of Sine-Gordon and KdV type. Many physical applications demand generation of these waves by periodic small amplitude external driving force. In order to get the waves of finite amplitude, starting from zero equilibrium position, one needs to choose the frequences of the drive to be close to eigenfrequences of the nonlinear media. Moreover, while the amplitude goes up, the resonance with driving frequencies should hold (this is called *autoresonance condition*). The well-known autoresonant effect provides this condition, if the rate of driving frequency modulation is not too fast [1]. We will prove this effect for the integrable equation $\vec{u}_t + P(\vec{u}) = \epsilon \vec{f}(x,t)$, where P is nonlinear differential operator in x and $\vec{f}(x,t) = \vec{f}(\theta_1, \dots \theta_n), \quad \theta_j = \kappa_j x - \nu_j t$.

The proof is based on finite-gap integration method and multi-phase averaging for integrable systems. The conditions for autoresonance behavior are found in terms of solutions of the Whitham equations with the rhs depending on $\vec{f}$. Note that this procedure is essentially multi-phase, which can produce some physically interesting effects [2].

References

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