Some properties and exact solutions of 'noncommutative' KP hierarchies

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If Z is any solution of the matrix linear system $Z_{t_n} = H^n Z$, $n = 1, 2, \ldots$, where

$$Z = \begin{pmatrix} X \\ Y \end{pmatrix}, \qquad H = \begin{pmatrix} R & Q \\ S & L \end{pmatrix},$$

with an invertible $N \times N$ matrix X, an $M \times N$ matrix Y, and constant matrices L, Q, R, S, then $\phi = Y X^{-1}$ solves the potential KP hierarchy in the algebra of $M \times N$ matrices with product $A \cdot B := AQB$. Here the matrices are taken over the algebra of complex functions of $t_1, t_2, \ldots$, but this result extends to matrices over any associative algebra with product $*$ by setting $\phi = Y * X^{*-1}$ (and requiring L, Q, R, S to satisfy certain compatibility conditions with $*$). If Q has rank 1, solutions of this pKP_Q hierarchy are mapped to solutions of the scalar pKP hierarchy and important classes of solutions are easily recovered in this way. Underlying the above construction of solutions of matrix pKP hierarchies is a *weakly nonassociative* algebra structure and a corresponding general relation with 'noncommutative' KP hierarchies. Here we present an elementary approach to this relationship and report on recent results, partly based on [1, 2].

References

[1] Dimakis, A. and Müller-Hoissen, F., *nlin.SI/0701010* and math-ph/0703067.

[2] Dimakis, A. and Müller-Hoissen, F., *J. Phys. A: Math. Theor.* **40** F321-F329, 2007.