Invariant polynomial subspaces

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In this talk we present some structure theorems which characterize the spaces of linear and non-linear differential operators that preserve finite dimensional subspaces generated by polynomials in one variable.

In the case of linear operators, these results apply to the theory of quasi-exact solvable finite-gap potentials. Let $\mathcal{P}_n$ be the vector space of polynomials of degree n or less. We are able to characterize all codimension-one subspaces of $\mathcal{P}_n$ that are invariant under a second-order differential operator that does not preserve $\mathcal{P}_n$. The exceptional polynomial subspaces lead to quasi-exactly solvable potentials outside the Lie-algebraic class[2]. We also obtain exact solutions for the band energies of a finite-gap elliptic potential outside the Treibich-Verdier class.

In the case of non-linear operators, a basis may be produced by means of the concept of deficiency. As a further refinement, we are able to describe all translation-invariant operators with quadratic non-linearity of a given deficiency. Time permitting, we will present an application to exact solutions of non-linear evolution equations. [3].

References

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