Peakons and shockpeakons in the Degasperis–Procesi equation

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The Degasperis–Procesi equation $m_t + m_x u + 3mu_x = 0$, where $m = u - u_{xx}$, is an integrable wave equation similar in form to the Camassa–Holm shallow water equation $m_t + m_x u + 2mu_x = 0$. These equations share some features, such as peakon (peaked soliton) solutions, but there are also important differences between them.

The Lax pair for the CH equation contains a classical second order self-adjoint spectral problem, whereas the corresponding problem for the DP equation is of third order and nonselfadjoint. In both cases, the solution of the inverse spectral problem gives explicit formulas for the multipeakon dynamics; the DP case involves some interesting generalizations of orthogonal polynomials and Padé approximation [**?**].

The DP equation admits less regular solutions than the CH equation, for example "shockpeakons" [**?**] where u has jump discontinuities (ordinary peakons have jumps in u_x but not in u). Shockpeakons form naturally at peakon-antipeakon collisions in the DP equation, in contrast to the CH case where peakons and antipeakons just pass through each other.

References

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