# Rank One Conditions and the Geometry of the Hirota Bilinear Difference Equation

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An investigation of the geometric implications of the Hirota Bilinear Difference Equation will allow us to answer two seemingly unrelated questions.

1. There seem to be many different places in the study of integrable systems that "rank one conditions" arise (for example, the Lax operators of Caloger-Moser and Ruijsenaars-Schneider particle systems, the construction of tau-functions for KP solitons in my 2001 paper with Gekhtman, and the "operator identity" construction of Sakhnovich). Is there a common source for all of these separate instances of this phenomenon?

2. The original paper of Sato on the link between Grassmannians and solitons suggests that each Plücker relation corresponds to a different difference equation. Since all of the Plücker relations are necessary to characterize a Grassmannian, how can the Hirota Bilinear Difference equation alone characterize KP tau-functions?

The answer to the first question (as published in a paper with M. Gekhtman that appeared in *J. Geom. Phys.* in 2006) reveals a "hidden" rank one condition in the original Sato construction: a rank one condition arises naturally if we consider more general flows on the Grassmannian and require them to generate KP tau-functions. The answer to the second question yields a simpler and, in some ways, more natural way to characterize Grassmannians than the usual Plücker relations. This unexpected application of the HBDE in algebraic geometry is to appear in *Proceedings of the AMS* in a joint paper with T. Shiota and two College of Charleston undergraduates.