Asymptotic solutions at infinity of the Hamiltonian systems

Leonid Kalyakin[1]

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1. Institute of Mathematics RAS. Chernyshevski str. 112, 450077 Ufa (Russia).

Nonlinear nonautonomous system of two ordinary differential equations is considered.

$$t^{-k}\frac{dr}{dt} = -\partial_\varphi H(r,\varphi,t) + F(r,\varphi,t), \quad t^{-k}\frac{d\varphi}{dt} = \partial_r H(r,\varphi,t), \quad (k\in\mathbb{Z}, k\geq 0).$$

It is supposed that the leading order terms of the equations at infinity have a hamiltonian structure and they are represented in the form of the action–angle variables.

$$H(r,\varphi,t) = H_0(r) + \sum_{n=1}^{\infty} H_n(r,\varphi)t^{-n}, \ F(r,\varphi,t) = \sum_{n=1}^{\infty} F_n(r,\varphi)t^{-n}, \quad t\to\infty.$$

The minor terms at infinity have a periodic dependence on the angle φ. We give an asymptotic analysis at infinity of two parametric solution:

$$r(t;r_0,\varphi_0) = r_0 + \mathcal{O}(t^{-1}), \ \varphi(t;r_0,\varphi_0) = \omega(r_0)t^{k+1}[1+\mathcal{O}(t^{-1})]/(k+1).$$

Such type results as applied to the main resonance equations solve the problem of autoresonance occurred in nonlinear systems, [1].

References

[1] Kalyakin, L. A. Justification of Asymptotic Expansions for the Principal Resonance Equations *Proc. of the Steklov Inst. of Math.*, **Suppl. 1** (S108–S122), 2003.