Geometric Properties and Inverse Scattering for the Camassa-Holm Hierarchy

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The squared eigenfunctions of the spectral problem, associated to the Camassa-Holm equation represent a complete basis of functions, which helps to describe the Inverse Scattering Transform for the Camassa-Holm hierarchy as a Generalised Fourier Transform (GFT). All the fundamental properties of the Camassa-Holm equation such as the integrals of motion, the description of the equations of the whole hierarchy and their Hamiltonian structures can be naturally expressed making use of the completeness relation and the recursion operator, whose eigenfunctions are the squared solutions. In the talk, using GFT we will describe explicitly some members of the CH hierarchy, including integrable deformations for the CH equation. Also we will show that solutions of some 2+1-dimensional generalizations of CH [1] can be constructed via the Inverse Scattering Transform (IST) for the CH hierarchy.

In geometric terms, CH is a geodesic flow equation on the Bott-Virasoro group, preserving the so-called H^1 metric [2]. This example is analogous to the Euler equations for a rigid body with a fixed point, describing geodesics for a left-invariant metric on SO(3). In relation to this we report how the recently obtained solutions [3] for the CH hierarchy lead to an explicit parametrization of the Virasoro group.

References

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