Solitary Waves in a Madelung Fluid Description of Nonlinear Schrödinger Equations

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Recently, using a Madelung's fluid description, a connection between envelope soliton-like solutions of NLS-type equations and soliton-like solutions of KdV-type equations was found and investigated by Fedele et al. [Eur. Phys. J. B, **27** (313 - 320), 2002; Physica Scripta, **65** (502 - 508), 2002]. A similar discussion is possible for the class of derivative NLS-type equations. For the completely integrable dNLS equation

$$i\alpha\frac{\partial\Psi}{\partial t}+\frac{\alpha}{2}\frac{\partial^2\Psi}{\partial x^2}+i\beta\frac{\partial}{\partial x}\left(\left|\Psi^2\right|\Psi\right)=0$$

and for a motion with stationary-profile current velocity, the fluid density ρ ($\rho=|\Psi^2|$) satisfies a stationary Gardner's equation.

$$\frac{\alpha^2}{4}\frac{\partial^3\rho}{\partial\xi^3}+\left(u_0^2+2c_0\right)\frac{\partial\rho}{\partial\xi}-3u_0\frac{\beta}{\alpha}\rho\frac{\partial\rho}{\partial\xi}+\frac{3}{2}\left(\frac{\beta}{\alpha}\right)^2\rho^2\frac{\partial\rho}{\partial\xi}=0$$

($\xi=x-u_0t$). If $c_0+\frac{u_0^2}{2}<0$, the obtained 1-soliton solution of dNLS equation is similar to the solution found by IST method [D. J. Kaup, A. C. Newell – J. Math. Phys., **19** (798 - 801), 1978] . The discussion is easily extended to a larger class of dNLS equations.