Extending Harmonically a Symmetric Map

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Let $\phi \colon \mathbb{S}^1 \to \mathbb{S}^1$ be a given boundary map. Then, a C^2 map $\Phi \colon \mathbb{H}^2 \to \mathbb{H}^2$ is said to be an extension of ϕ if it is continuous up to the boundary and it coincides there with ϕ. It was conjectured by Schoen in [1] that every quasi-symmetric homeomorphism of the circle can be extended to a quasi-conformal harmonic diffeomorphism of the hyperbolic 2-disk into itself, and that such an extension is unique. The set of symmetric self maps of the unit circle forms a big class among all the quasi-symmetric ones. The goal of this paper is to prove that if a quasi-symmetric homeomorphism of the unit circle is symmetric then it can be harmonically extended to a quasi-conformal map. This has been conjectured by Zhong Li in [2] to be true. A proof that this conjecture is valid first appeared in [3] by Markovic using the theory of Theichmüller spaces and bounded holomorphic differentials. In this paper the same result is established with a proof applying the compact exhaustion and PDE methods.

References

[1] R. SCHOEN. The Role of Harmonic Mappings in Rigidity and Deformation Problems. *Complex Geometry, Lecture Notes in Pure and Applied Math.*, **143**(179-200), 1993.

[2] Z. LI. On the Boundary Value Problem for Harmonic Maps of the Poincaré Disk *Chinese Science Bullentin.*, **42**(591-635), 1993.

[3] V. MARKOVIC. Harmonic Diffeomorphisms of Noncompact Surfaces and Teichmüller Spaces *J. London Math. Soc.*, **65**(103–114), 2002.