Investigating knot types of finite-gap solutions of the Vortex Filament Equation

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The simplest model of vortex filament motion in an ideal fluid leads to an integrable nonlinear evolution equation, the Vortex Filament Equation (or Localized Induction Equation), which is closely related to the cubic focussing nonlinear Schrödinger equation. Under periodic boundary conditions, the finite-gap solutions of the Vortex Filament Equation [2] provide interesting examples of evolving knots whose geometric and topological features can be related to their algebro-geometric description (a hyperelliptic Riemann surface and a divisor satisfying certain reality conditions).

In this talk, we will address the extent to which topological information can be inferred from the algebro-geometric data. In particular, we will describe how the theory of isoperiodic deformations (developed by Grinevich and Schmidt [1], after Krichever) can be used to generate a family of closed finite-gap solutions of increasingly higher genus close to multiply covered circles. Each step of the deformation process corresponds to constructing a cable on the previous filament, whose knot type is determined from the deformation scheme, and is invariant under the time evolution [3].

References

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