Extended resolvent approach to Inverse Scattering in multidimensions

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May 1, 2007

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We study the KPI and KPII equations that can be considered as two prototypes of integrable equations in 2+1 dimensions. They are associated respectively to the nonstationary Schrödinger operator and to the heat operator.

KPI and KPII, being two generalizations of the KdV equation, admit solutions behaving at space infinity like the solutions of the KdV equation. However, any effort of building Inverse Scattering for solutions with constant behavior along some rays in the plane clashes with divergency of integral equations defining the Jost solutions of the Schrödinger and heat equation.

One needs to explore the nature of the associated linear operators by considering the whole of their Green's functions in a very general class. In our language this corresponds to considering the extended resolvent of both operators.

Then, one can proceed in two successive steps: first, by considering the pure N soliton solution and, afterwards, by adding an arbitrary smooth decaying background. Precisely, first, we get the explicit form of the extended resolvent of the pure N soliton potential and, then, we introduce the perturbation dressing this resolvent. We are able to deal successfully with the singularities due to the constant behavior of the potential at large space and to solve completely the direct and inverse problem for KPI and KPII.

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