

Interaction of Domain Walls in an Anisotropic XY Model

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The dynamics of magnetisation in the anisotropic XY model is described by the “repulsive” (or “defocusing”) parametrically driven nonlinear Schrödinger equation,

$$i\Psi_t + \frac{1}{2}\Psi_{xx} + (1 - |\Psi|^2)\Psi = h\Psi^*.$$

The Bloch and Ising walls are two coexisting stable soliton solutions of this equation. We study the interaction of the two walls by following the dissociation of an unstable Bloch-Ising bound state. Mathematically, the analysis focusses on the splitting of a four-fold zero eigenvalue associated with a pair of infinitely separated Bloch and Ising walls.

The lifting of the degeneracy of a repeated eigenvalue of the scalar Schrödinger operator with the potential comprising two identical potential wells with large separation, is discussed in standard textbooks on quantum mechanics. This analysis is not helpful in our case, unfortunately, for three reasons: (i) our linearised operator operates on vector, not scalar, functions; (ii) the potential wells formed by the Bloch and Ising walls are not identical; (iii) the textbook analysis *postulates* a particular form of the wavefunction on symmetry grounds, rather than deriving it within some perturbation formalism — as a result, the generalisation to the vector nonsymmetric case is not straightforward.

Our treatment is based on expanding the eigenfunction in the asymptotic series near the cores of the two walls and matching the resulting expansions in the overlap region.

We show that a Bloch and an Ising wall interact as two classical particles, one with positive and the other one with negative mass.