Boundary Algebra and Exact Solvability of the Asymmetric Exclusion Process

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In the matrix product states approach to nonequilibrium physics the stationary probability distribution of the asymmetric exclusion process is expressed as a matrix product state with respect to a quadratic algebra. The quadratic algebra defines a noncommutative space with the $SU_q(2)$ quantum group action as its symmetry and this is the bulk symmetry of the model. Boundary processes amount to the appearance of parameter dependent linear terms in the algebraic relations and lead to a reduction of the $SU_q(2)$ symmetry. We find the boundary quantum group of the asymmetric simple exclusion process to be a tridiagonal algebra which is the linear covariance algebra for the bulk $U_q(su(2))$ symmetry. The irreducible representations of the boundary algebra are given in terms of the Askey-Wilson polynomials whose four parameters are related to the boundary rates. The Askey-Wilson algebra arises as a hidden symmetry and allows to solve the model exactly.