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nonlinear evolution equations and dynamical systems

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An introduction to pattern formation
Prof. **Alastair Rucklidge** (Leeds University)

The transition from regular to irregular motion as travel on Riemann surfaces
Prof. **Paolo M. Santini** (Università di Roma “La Sapienza”)

Properties of low dimensional dynamical systems in the large
Prof. **Carles Simó** (Universitat de Barcelona)

Synchronization and networks
Prof. **Steven H. Strogatz** (Cornell University)

An introduction to pattern formation

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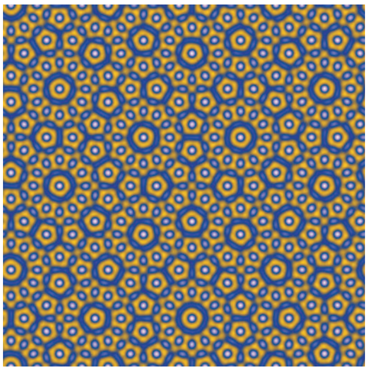


Figure 1: Example of an approximate quasi-pattern in a periodically forced PDE

Lecture 1: Stability theory and weakly nonlinear theory

In this lecture, I will introduce different examples of pattern formation, and start the analysis with linear theory of isotropic systems. There are four types of instability: the growth rate can either be real or complex (leading to steady and oscillatory patterns), and the preferred wavelength can either be finite or infinite. The simplest case is that of a finite wavelength instability with real growth rate. We will discuss the nonlinear evolution of this case, using a classic model of pattern formation, the Swift–Hohenberg equation.

Lecture 2: Pattern selection

We next turn to the question of pattern selection: what is that makes stripes the preferred pattern in some situations, while in others, squares or hexagons (or more complex patterns) might be preferred. We pose this question under the assumption that the patterns are periodic in space, which leads naturally to an exploration of the importance of symmetry in the question of pattern selection.

Lecture 3: Pattern formation in large domains

If the domain under consideration is large compared to the pattern's preferred wavelength, there are further possibilities. I will discuss some of the issues behind the formation of spirals (and spiral defect chaos), as well as long-wave instabilities of patterns, and the small divisor problem in the formation of quasi-patterns.

The transition from regular to irregular motion as travel on Riemann surfaces

Paolo M. Santini (Università di Roma “La Sapienza”)

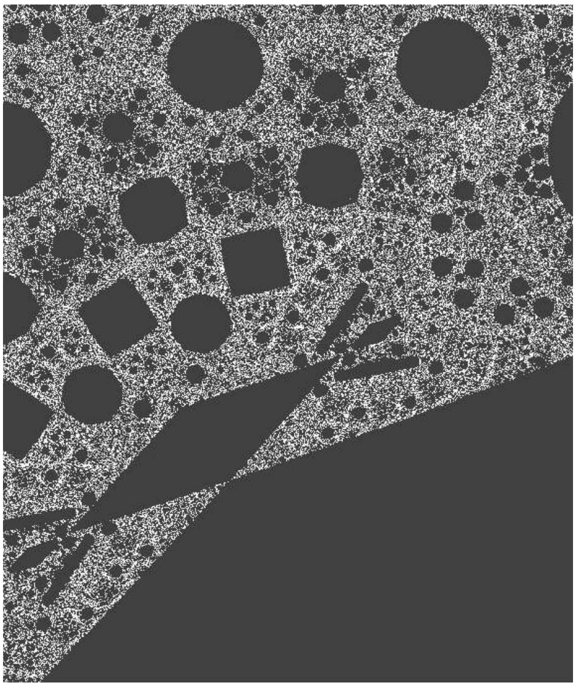


Figure 2: A detail of the phase space of the center map, for $n = 16$ and $R = 0.31$, generated by $481 \cdot 10^6$ iterations and magnified 300 times.

It is well-known that the degree of integrability of a dynamical system is intimately related to the analyticity properties of its solutions in complex time. For instance, for Kowaleski, Painlevé and his school, the request that the only movable singularities are poles has been a very succesful tool to isolate and classify important integrable cases. The idea that integrability is compatible even with the presence of movable branch point singularities, provided they are not “too dense”, is more recent. In these lectures we explore the effects of movable branch point singularities on the dynamical properties of evolutionary systems, in particular, in connection with the transition from regular to irregular motions with sensitive dependence on the initial conditions. We plan to illustrate these notions on some examples, including the Calogero many-body model with rational interaction and two complexifications of dynamical systems reducible to quadratures.

Lecture 1: Local analysis and movable singularities

Fixed and movable singularities of a system of ODEs; Painlevé test and Painlevé equations. Case of a finite number of movable branch point singularities: the Calogero-Moser many-body problem and its Riemann surface. Branch points versus particle collisions.

Lecture 2: Branch point singularities and irregular motion

Case of movable branch point singularities dense on a curve of the complex time plane: the Aristotelian three-body problem. Calogero's trick, cyclic dynamics and isochrony. Rational and irrational coupling constants of the model versus algebraic and non-algebraic Riemann surfaces.

Lecture 3: How complexification can generate complexity

Case of movable branch point singularities everywhere dense in the complex time plane: the complexified anharmonic oscillator $\ddot{z} = -dz^n/dz$, $n \in \mathbb{N}$ and the basic properties of its Riemann surface. Cyclic dynamics of radius R, and their equivalent description: the (n, R) - center map: isochrony and fractals.

Properties of Low Dimensional Dynamical Systems in the Large

Carles Simó (Universitat de Barcelona)

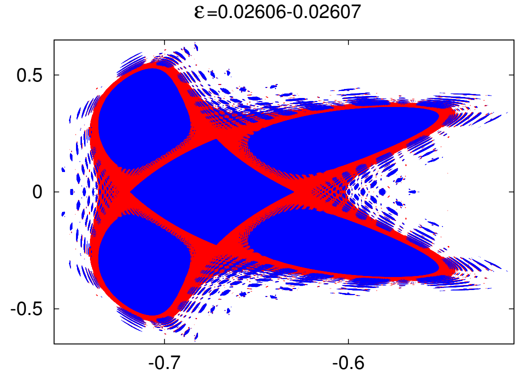


Figure 3: The Kuramoto-Sivashinsky PDE $u_t + \nabla^4 u + \nabla^2 u + \frac{1}{2}|\nabla u|^2 = 0$ has travelling wave solutions which in the 1D case are of the form $u(t,x) = v(x-ct)$ and satisfy the so-called Michelson system $w''' = c - w' - \frac{1}{2}w^2$, where $w = v'$. This is a conservative system in $\mathbb{R}^3$. With suitable space and time scalings can be converted to $X''' = -1 - \varepsilon^{-1/3}X' + X^2$. A standing question is to know the set of orbits of this system which are bounded for all time. To this end a Poincaré section through $X'' = 0$, $X''' > 0$ has been introduced and (X', X) are used as coordinates, after a scaling of X'. For $\varepsilon = 0.02606$ essentially all the points shown in red or blue subsist. For $\varepsilon = 0.02607$ only the points in blue. Invariant curves, which exist for $\varepsilon = 0.02606$, have been destroyed when increasing ε. Points in red and not in blue mostly belong to a bounded chaotic zone and are able to escape when the confining curves are destroyed. Many tiny chains of periodic islands can be seen, both around the large period-4 islands and close to the curves to become destroyed. The mechanisms are universal. Furthermore, Michelson system is a paradigm in the conservative unfoldings of the Hopf-saddle-node bifurcation.

The objective of these lectures is to introduce to the study of systems having a moderate number of dimensions and/or parameters.

The main goal is the study in large parts of the phase space times the parameter space. Several simple paradigmatic models will be presented and analyzed by means of analytic and numerical tools. This analysis will concern:

Lecture 1. The problems of stability in the large

The problems of stability in the large. Presentation of a list of 8-10 models, from the conservative Hénon map to the motion of giant planets in the solar system, with possible applications. Details on 2 or 3 of these models. Numerical simulations and statistics of results. The mathematical challenges appearing to explain the phenomenology.

Lecture 2. Analysis of some mathematical problems

Some mathematical problems: destruction of invariant curves, chains of transversal heteroclinic conenctions, scaling and universality. Simple paradigmatic models: the standard map, separatrix and biseparatrix maps. Analysis and results. Going back to the problems of Lecture 1. Tools to pass from numerical evidences to rigorous proofs.

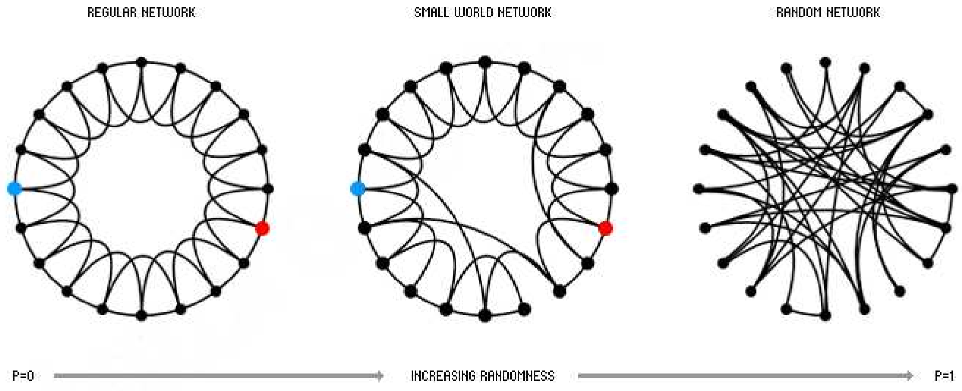
Lecture 3. Effective tools for the detection of the dynamics

Some effective tools. Taylor integration method: description, properties and suitability, effect of round off errors. A sample of examples. Detecting the character of the dynamics: Lyapunov exponents and other indicators. Computing relevant objects: symbolics and numerics. Poincare maps, periodic solutions, invariant manifolds, invariant tori, homoclinic and heteroclinic connections.

Conclusion and perspectives: Towards a global study of the dynamics. The problems in higher dimension.

Synchronization and networks

Steven H. Strogatz (Cornell University)



This series of elementary lectures is intended for scientists and mathematicians who want to learn the basics of complex networks and self-synchronizing systems. The first two lectures are expository and entertaining, and hopefully will stimulate the listener to delve more deeply into the theory behind the subjects. The final lecture is a bit more technical, and will be presented at the blackboard, so that every step can be followed easily.

Lecture 1. Sync: The Emerging Science of Spontaneous Order

What caused hundreds of Japanese children to fall into seizures while watching an episode of the cartoon show Pokemon? Why do women roommates sometimes find that their menstrual periods occur in sync? The tendency to synchronize is one of the most mysterious and pervasive drives in all of nature. Every night along the tidal rivers of Malaysia, thousands of fireflies flash in silent, hypnotic unison; the moon spins in perfect resonance with its orbit around the Earth; the intense coherence of a laser comes from trillions of atoms pulsing together. All these astonishing feats of synchrony occur spontaneously – almost as if the universe had an overwhelming desire for order. On the surface, these phenomena might seem unrelated. After all, the forces that synchronize fireflies have nothing to do with those in a laser. But at a deeper level, they are all connected by the same mathematical theme: self-organization, the spontaneous emergence of order out of chaos. Video footage of synchronous fireflies, and the notorious crowd synchrony that triggered the wobbling of London's Millennium Bridge, will be shown.

Lecture 2. Six degrees of separation: Small-world networks in science and society

Everyone is familiar with the small-world phenomenon: soon after meeting a stranger, we are often surprised to discover that we have a mutual friend, or that we are somehow linked by a short chain of friends. In this talk, I'll present evidence that the small-world phenomenon is more than a curiosity of social networks – it is actually a general property of many networks found in nature and technology, ranging from nervous systems to the power grid and the Internet. I'll also speculate about some of the broader implications of these findings (e.g., for the spread of infectious diseases), and will reveal the identity of the actor at the center of the Hollywood universe (it's not Kevin Bacon). This is joint work with Duncan Watts

Lecture 3. Infinite-Order Phase Transition in a Randomly Grown Network

Networks are all around us, from the World Wide Web to the gene networks inside our cells. In many cases, these networks change over time, as nodes and edges are added, deleted, or rewired. The math problems raised by such evolving networks are of interest, not just to graph theorists and computer scientists, but also to people working in dynamical systems and statistical physics.

In this talk, I'll discuss what may be the simplest model of a randomly growing network. At each time step, a new node is added; then, with probability δ, two nodes are chosen uniformly at random and joined by an undirected edge. This process is repeated for t time steps. In the limit of large t, the resulting graph displays surprisingly rich characteristics. In particular, a giant connected component emerges in an infinite-order phase transition at $\delta = 1/8$.

No knowledge of graph theory is needed to follow this talk; the main ideas come from elementary probability and differential equations. This is joint work with Duncan Callaway, John Hopcroft, Jon Kleinberg, and Mark Newman.